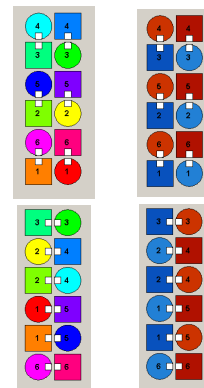


The Contra Cube

Chris Page's article *The twenty-four dupe minor formations* gives a very clear classification of the 24 possible arrangements of dancers on hands four spots and the transitions between them. These arrangements fall into three natural families *Improperish*, *Properish* and *Diagonal* which are closed under the commonly used symmetric choreography.

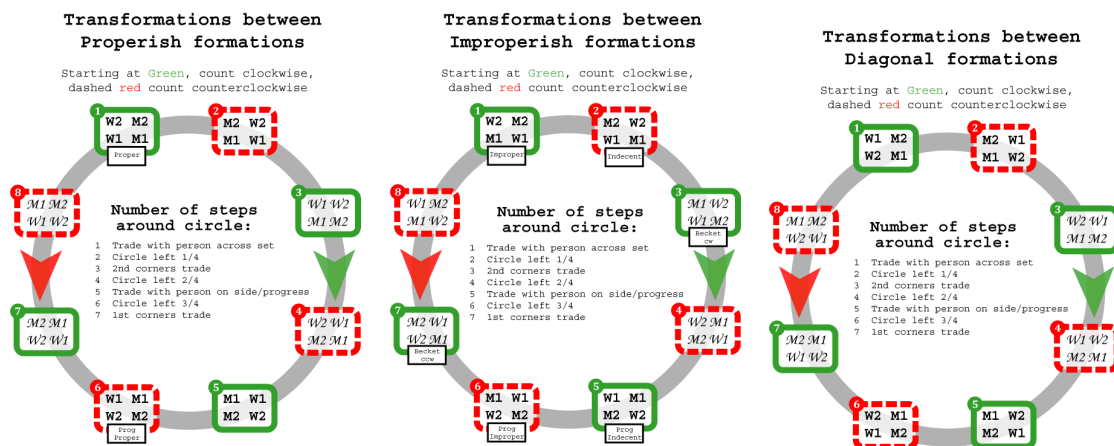
Formations

The formations described in Chris' article are permutations of the symbols M1, W1, M2, W2. An equivalent view point in the initial diagramming of a dance uses the same color for all dancers of the same type. To the right is an improper set up with six couples and a coloring, so the arrangement of colors in each of the hands four stations look identical.



Part way thru the dance (which has some out of set action) the dancers find themselves as pictured, but all hands four stations have colors arranged the same and the coloring has 'circled left 1/4'

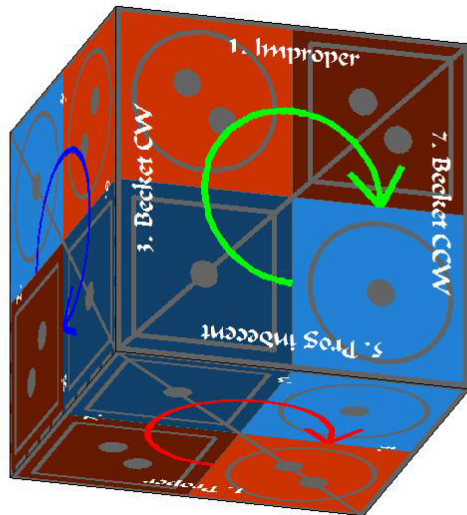
Examining a specific hands four station, how the four labels or colors move give the permutation corresponding to a given dance move¹. Chris produced following three figures classifying what might be observed and showing how contra dance moves 1 to 7 give transformations between the eight formations.




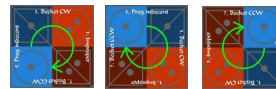
¹ A similar idea was used to understand grid squares where a coloring of the eight dancers in the squares gives an induced permutation.

Rotating a “Contra Cube” give Chris’s formations

Rotations of a cube allow the formation families and transitions between them to be visualized. The opposing faces relate to the solid and dashed formations in Chris’ diagrams. To use this cube set the initial

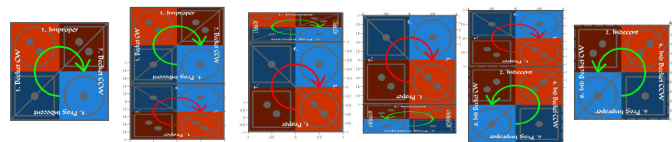


formation uppermost:  The seven contra dance moves are all rotations of the cube. The circling moves (2,4 and 6) are just the same circling of the cube around a vertical axis finishing at:



respectively.

A trading move such as 1 (trading across the set), is found by using a rotation axis across the cube giving



Note that half way through a trading move the top face of another formation family appears. See the video at: <http://youtu.be/zVA00791deg>

Solid and Dashed Lines vs. Clockwise and Anticlockwise Oriented Faces

In Chris’s diagrams the formations outlined with solid and dashed lines correspond opposite faces of the cube. The faces of the cube are given an orientation (shown by the red, green and blue circular arrows) by the rotation axes through each face. A simple renumbering of Chris’s Diagonal family diagram gives solid and dashed corresponding to clockwise or anticlockwise oriented faces, resp.

A and B subfamilies vs. First and Second Corner families

Chris’s distinguishes between **A** and **B** formations. For Improperish and Properish, **A** signifies partners across from each other, and **B** partners on the side (i.e. Becket). For the Diagonal formations, it has no obvious characterization. For the cube rotation model, the families divide nicely into two groups depending on the visible face of the cube have a diagonal of the tetrahedron on the first (\) or second (/) ‘corners.’

The transformation diagram between the families in which an asymmetric pairs of dancers are swapped has a certain asymmetry with regards to **A** and **B** these are switched only between the Improperish and Diagonal families. However types (\) and (/) always switch when moving between any two different families.

First Member of Each Family

The choice of the beginning of each family is somewhat arbitrary, and Chris has chosen the most common Improperish and Properish formations as the first of each. Mathematically it would be nicer if the Properish family started with #3 as then all families would start on a clockwise 2nd corner diagonal face ☺.

Danceable transitions

Chris's paper discusses 3 types of transforms between families. The final two are the most simple in the cube model: the asymmetric moves *Square Root of Trade Up/Down* and *Square Root of Trade Across*. They are just rotating the cube $\frac{1}{4}$ around the two axes parallel to the top face. These move all four dancers, but when done twice gives a trade (5 and 1, resp.). The first two involving one non-diagonal pair of dancers trading² are combinations of the cube rotations. For example "*Circle Left 1/2, Square Root of Trade Across*" trades across the pair of dancers closest to the top of the set. Similarly the four involving three dancers circling can be given as combinations of cube rotations.

Examples:

Four in Line: Chris observed that having 1s step between the 2s can be viewed as Diagonal. In the usual improper hands four, having the facing couples pass thru, either the usual way or 1s stepping between the 2s, just before the dancers are all in a line of 4 if some dancers just get slightly ahead, one pair trades first, then shortly after the second trade is made. So in a sense this is the circle across move being done twice in a very short time frame.

4 face 4 is two parallel contra lines. Center or end directed moves are symmetric with respect to the overall formation, but asymmetric in each contra line half. *Center two pass thru and run around the ends* (MWSD terminology) on each line is an asymmetric move (of the type quarter double figure eight³) and puts each half into Diagonal formation. Doing this twice gives box the gnat across (= pass thru).

A template for a 4 face 4 dance with each half in Diagonal formation:

A1 (middle 4) LH star $\frac{1}{2}$, (diag) Nbr Right Allemand $\frac{1}{2}$, Center Left Allemand 1 $\frac{1}{2}$
 ... play in diagonal formation keeping formation at end of A1
 ... centers pass thru, Partner Swing face your direction

Contra-like Dances

² Mathematician John Conway invented a two couple dance with rope using just circling together with one such move in which the ropes become entwined and then untangled.

³ These moves were looked at in the context of MWSD. They are the four asymmetric two couple moves (among a total of ten moves) that can be used in a generalization of the Chicken Plucker routine.

Mixer “contra-like” dances. The formation structure applies to dances that look like a contra dance, but don’t keep partner or don’t progress. This implies that the progression aspect is not covered by this formation model.

Here men and women progress at different rates, but the formation looks like a contra:
 A1: N AR 1½ (next) N AL 1½ A2: LC, Bal sqth 2 B1: Sh B&SW B2: LC, Bal Sqth 2
 5 5 5 3 4 5 5 3 4 5

555 34 55 345 in Chris’s notation, note Sqth2 is 4 or 15

Some Dance (ideas)

Sync or Swim

Cross Contra Formation (Reverse Progression)

(set up from improper, 1s between 2s, face N and step to RH long waves)

A1 (opposite sex) Neighbor Right Allemand (any multiple of 1/2)(8);

Balance the Wave (4); Circulate (in facers cross, outfacers loop R)

A2 Balance L/R ROM (8) Bal R/L ROM (8)

B1 (person in RH) Right Allemand ½ (4), Circulate (4); Neighbor Swing

B2 Four in Line Down the Hall, (outsides, 1s, slow, insides, 2s step in between next)
 cast to turn as couples, return, (face and step to RH waves)

Notes

1. If both couples are in sync doing the LA in A1 and go the same amount, the formation will stay diagonal and interactions at end of A1b and A2 will be with same sex neighbors. If the amount differs the formation will become Properish and the interactions will be with partners.
2. So this dance can be started improper long wave men face out, women in. The dancer could even not be told anything about the formation and see if they notice. If dancers always stay synchronized in this case it would never become diagonal. Could start diagonal and have A1 start N AL 1 ½ always.
3. The ROM could be done R/L then L/R but interaction with future and past N as written
4. Self Critique
 - a. Still a bit too clockwise for my taste.
 - b. Arbitrary choice of Allemand amount in A1 but in A2b allemand must be precise - hard to remember?
 - c. **Very problematic end effect**, since in B2 the 2s stepping up to next are NOT normal i.e. Lady is on Man's LEFT this requires the neutral couple at the top to stay that way, but because B1b ends in a swing, many neutrals may swing each other and then finish normal.
5. The title refers what happens with partners if they didn't synchronize the AR starting the dance – is a ROM like swimming?

Men look to the Future, Women to the past

Cross Contra Formation – Reverse Progression

(set up from improper, 1s between 2s, face N and step to LH long waves)

A1 (opposite sex) Neighbor Left Allemand 1; Balance, Circulate (loop L)

A2 Bal R/L ROM; Bal L/R ROM

B1 (same sex) Neighbor Left Allemand 1 3/4 (so) 2s Swing (all face down)

B2 Four in Line Down the Hall, (centers California Twirl, ends move to next and run around the centers as they spread apart) to return, (face and step to LH waves)

Note 1. At A2 all have same sex N in RH, Men have next same sex N in LH, Women have previous same sex N in LH. The end effects are easy the dancers out, stand normal and face back in.

Fatal Attraction Variation of Cary Ravitz's dance.

This uses the 4 in line method to get into diagonal formation and reorders Cary's dance to start in diagonal

Cross Contra Formation – Forward Progression

(set up from improper, 2s between the 1s, face N and step to LH long waves)

A1 Cary's "Meet and Loop Move"⁴

A2 Neighbors balance and swing

B1 Circle left 3/4; Partners swing (face down the hall)

B2 Four in Line Down the Hall, (left couple, 1s, step up to new couple then)

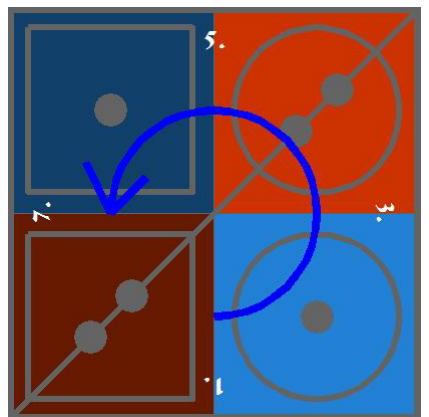
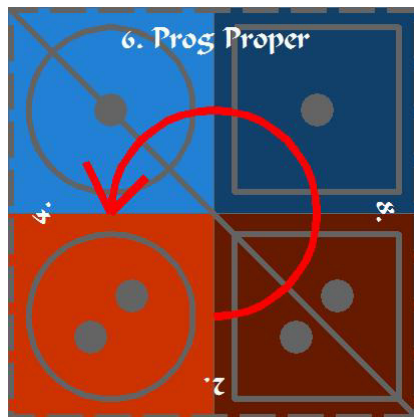
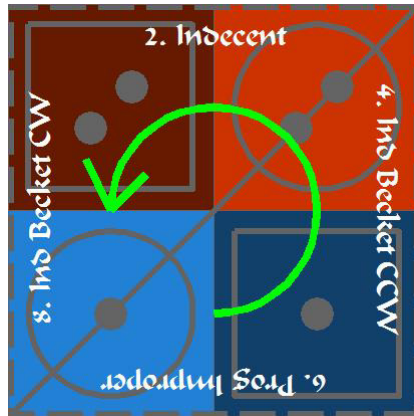
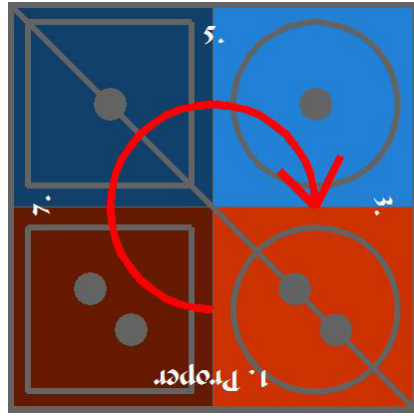
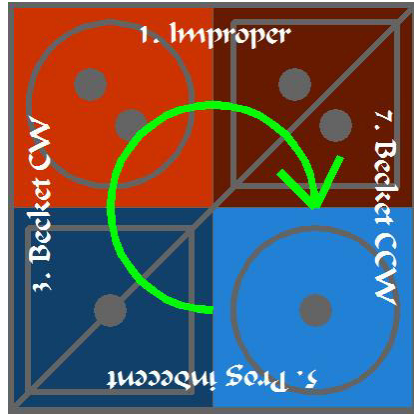
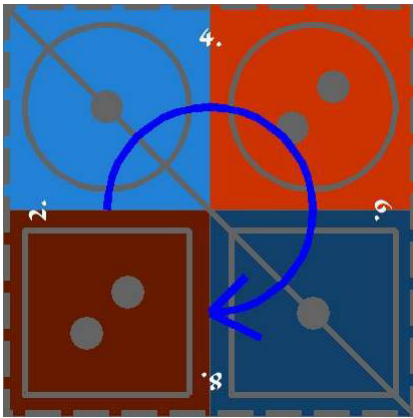
Ladies swap (Gent RH high, LH low), return, (face and step to LH waves)

Note

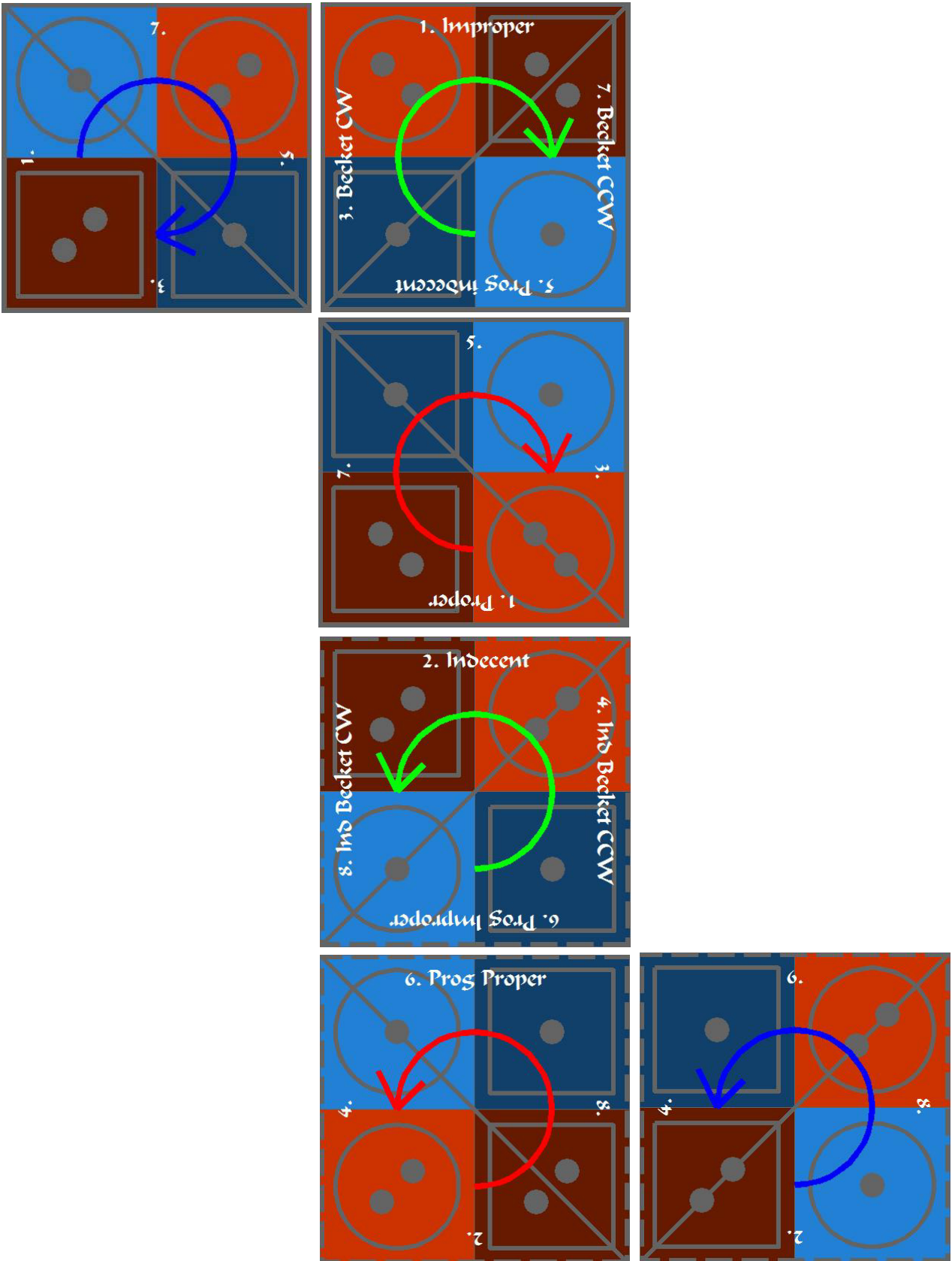
1. Can Slide Left after CL 3/4 in B1 then catch up in B2 not needed.
2. B1 alt: Give and Take (to gent's side), Partner Swing
3. Can set up in RH waves and do mirror of Cary's push and loop. Then A2 Neighbor Gypsy and Swing nice.

⁴From <http://ravitz.us/dance/#fa> Note your place in the set of four. Each 4 counts you will move one place to the right in the set. Face your partner and lock eyes. Ones meet in middle and back up to the next place, twos loop out to the next place (4). Twos meet in middle and back up to the next place, ones loop out to the next place (4). Ones meet in middle and twos loop (4). Twos meet ones loop (4).

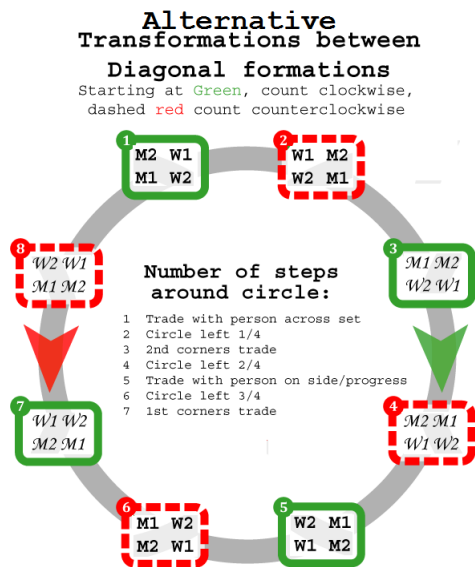
Make Your Own Formation Cube – Chris’s Scheme



Make Your Own Formation Cube – Modified Scheme



Renumbering of Diagonal Diagram



Maths

Rotations of the Cube are isomorphic to S_4

Contra dance formations are permutations on four elements or elements of the group S_4 . The three families correspond to three conjugate subgroups of order 8 each isomorphic to D_8 , the Dihedral group of the symmetries of a square.

Geometric rotations of a cube (about three types of axis giving 4-fold, 3-fold and 2-fold rotation) are isomorphic to S_4 , since the four main diagonals that connect opposite vertices are permuted under such rotations. The Contra Cube with faces colored by dancers has its final position the various formations. Since rotations are continuous, it is possible to move points representing dancers continuously through space. As the cube is rotated along its various axes, the points of intersection⁵ of these lines with the plane⁶ of a given face move thru the various permutations of the three families.

⁵ Another method of getting points on a square chose one end of each diagonal. Embedded in the cube are two tetrahedrons. Choosing the points of one of them give four points. Looking at the cube from the top, left side, or front, the shadows cast are the corners of a square. As the cube is rotated along its various axes, these shadows move thru the various permutations of the three families. However the correspondence is not so nice, mathematically it is not an isomorphism.

⁶ In the matlab simulations, the rotations are shown dynamically, but the intersection of the moving diagonal line goes to infinity as the line becomes close to parallel with the face, and then returns from the opposite direction! Stereographically projecting these points in the plane onto the sphere tangent to the center of the cube and passing through the top four vertices give a compact dynamic view. This sphere has radius $3/2$.

The family preserving and family changing moves

The period four rotations about the 3 axes perpendicular to the faces (colored red, green and blue) generate all the needed moves both to stay within a family⁷ and all moves between them. Letting r , g , b correspond to the $\frac{1}{4}$ rotation in the directions indicated on the face of the cube. For example if the green clockwise marked face of the cube is on top, bb , g , grr , gg , rr , ggg , rrg give Chris's moves 1, 2, ..., 7 resp. The non-diagonal transitions moves which trade two dancers are bgg , ggb , rgg and ggr . A transition moving just 3 dancers is gr , while a final type is r and b . Here the moves are describe with respect to the orientation and coloring of the cube, but an external (caller's) view point using moves like *Circle Left*, *Square Root of Trade Across*, *Square Root of Trade Up/Down*, *Trade 1st corner*, *Trade 2nd corner* as shown in the video.

Another viewpoint is that the commands of a contra dance cause dancers to move on the floor also induces commands for how a cube should rotate in space.

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⁷ The three conjugate subgroups are $\langle r, g^2, b^2 \rangle$, $\langle g, b^2, r^2 \rangle$ and $\langle g, r^2, g^2 \rangle$. Let $m = rg$, $m^3 = 1$. Checking $mrm^{-1} = g$, $mgm^{-1} = b$ and $mbm^{-1} = r$ shows they are conjugate.