Grid Squares

by Bill Baritompa (contra.baritompa.com)

Black = discussion, Blue maths, Green for imagined dance helpers.

Grid Squares are a type of contra dance with squares in a grid pattern, at each cycle dancers interact with their neighbors in the square and then move with partner to a new square. For the purpose of understanding the basic patterns, the dance is viewed as a rule or permutation that rearranges the couples on the floor in a fixed way, and is described in terms of simple permutations like circle right ¼, etc. which move dancers in each square as in a normal dance (i.e. have flag pole symmetry), and a few simple moves which move dancers between squares.

A set D of dancers (couples) is partitioned into sets of squares made of 4 couples. The square dance group SD is a subgroup of the symmetric group S_8, permutations on a set of size 8, that represents square dance moves[1]. Imagine the eight dancers numbered 1, … ,8 anticlockwise. In the diagram below the eight elements are {red man, red woman, green man, woman, cyan couple, purple couple}. “Circle right ¼” c = (1357)(2468), “heads right and left though” r = (15)(26). SD=centralizer(c^2) in S_8, all elements that commute with c^2 and so exhibit the required symmetry. It is a group of size 8x6x4x2 = 384. The subgroup that keeps couples together CG is <r, c> and is isomorphic to D_4, the dihedral group of order 8, the symmetries of the geometric square.

For grid squares we consider a group GrSD of permutations of D. Using the same notation, let SD and CG be the subgroups of regular permutations acting on each square. Let GrSD = <SD, h, s> where h and s are order 2 permutations which correspond to Heads (Sides) California twirl and pass through to the new square if you can, otherwise california twirl again. In figure 1, h rotates each block of the 4 cyan and red dancers between the squares about their central point, while leaving the rest unmoved. Similarly GrCG = <r, c, h, s>. Let φ: GrCG → CG be the homomorphism[2] where φ(g) = the word describing g with h’s and s’s removed. Other elements will be introduced as needed. We let d represent the dance permutation. The cyclic subgroup <d> acts on D. **Note** we use algebraic notation, so d acting on dancer x is denoted xd.

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[1] The Art of Square Dancing: Math in Motion by Christina Lorenzo (Senior Honors Thesis)

[2] Seems to be the case for h and s as described when grid consists of full squares, and fails otherwise. Used GAP to check many cases. However many permutations a and b, not nicely related to the coloring, also produce a homomorphism from <SD, a, b> onto SD. Since h and s behave nicely wrt the coloring of figure 1, from a direct argument I can show φ well defined. There must be some deeper reasons!

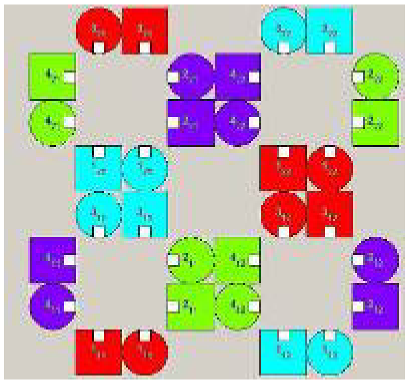


Figure 1

Hi Dancers, take your places on the floor for a Grid Square! Humor me by wearing a label the color of the spot you are standing on, and the location of your square. I'll walk thru the dance in a minute. You will dance in your squares awhile, and as in a square dance meet different kinds of neighbors: corners, opposites, left hand men etc, but like a contra you will move on to new squares with your partner to meet new neighbors.

Before we begin – any questions?

FAQ

Q1: (red man): That woman over there, will she ever be my opposite?

A: Sorry no, your opposite now has a cyan label so all your opposites will have that label color. That women's label is a different color ☹. By the way, not all cyan labeled women will necessary be your opposites. (Elements of GrCG act on D, φ(g) gives an induced action of the four colors. Opposite colors remain fixed.)

Q2: (purple woman) Will that man over there, ever be my corner?

A: Sorry no, his label is green the color of your current opposite man. Your corners will always have labels cyan or red. (Depends on φ(d), if an even permutation her corners will all be red, if an odd permutation will alternate between red and cyan.)

Q3: (purple woman again) Can't you tell me more about that man with the cyan label?

A: Well, the dance we will do is "**chscchscchs**" which doesn't change the orientation of the couples in a square, so in that case all your corners will have red labels. Too bad ☹. (φ(chscchscchs) = c).

Q4: Hey! Enough negativity, can't you say something positive?

A: Ok – you will keep your partner each time ☺.

Q5: Very funny! In a contra dance I get back to the same place after a certain time, also I sometimes have shadows. Does this happen in a grid square?

A: Yes, but in a contra dance when you return to the same spot everyone else does too. In a grid the frequency you return to the same spot varies. Also you can have a "shadow like" neighbor, that you might see every 4th time.

Q6: Can you explain more?

A: Ok, but let's do one walk through. Please humor me some more and make note of where you are now, so you will be able to go back there when I ask. So …

Some concepts used by mathematicians will be helpful here, but they can be described easily. So imagine that the dancers have had their walk through and are standing on their new locations. They have done one step of the permutation which encapsulates the whole dance into the future.

Q7: How did we do?

A: Great! Good job! It's no harder than a contra dance. Keep an eye on where you are now, but go back to your original spot. Great. Make a note of who's standing where you just came from. I want to take a little break from this dance, and do a 'scatter mixer.' We probably can do it as a no walk through. Just remember in this dance "couple you saw" refers to the ones I asked you to make a note of! Band: four potatoes for something old timey, please.

Make Orbit Mixer[3] (traditional)

Promenade your partner, go right now,
Around the floor any old how,
But if you find that "couple you saw"
Put 'em on your right with an old hee-haw,
Then promenade in lines of four
Adding "couples you saw" from off the floor.
(Or maybe they add you to them,
One is the leaf the other the stem.)
Dance like this until you're all in rings,
And "the dance is thru" the caller sings.

The cycle structure of d is the orbit description. It can be found very easily.

At this point the couples are in circles of various sizes around the room. In each circle the couples are those that occupied the other's spots in the grid dance after one time through the dance, and in the same order. These circles are the *orbits* of the dance permutation. Together they partition all the couples[4]. The sizes of the orbits and number of each size aid the understanding of the permutation.

Let the orbit decomposition of d be $\{O_1, O_2, \ldots, O_K\}$. The unique cycle decomposition is essentially the orbit decomposition. The number of dancers $N = |O_1|+|O_2|+ \ldots +|O_K|$.

The orbits can also be viewed as partitioning the spots on the dance floor. Although the orbit consists of distinct locations, we can imagine them connected by lines to show the paths[5] that the dancers move along from one cycle of the dance to the next. Each dancer belongs to exactly one orbit, so if orbit membership is used to color the dancers, some aspects of the dance simplify.

Those rings you are now in are called *orbits.* Each dance will have a distinctive collection of orbits. The total number of orbits as well as their sizes will say something about how the dance will feel. You've been very patient so far. Thanks. Before you go back to your original places on the floor (you still remember, don't you). Please do me one more small favor?

Q: Do we have to?
A: Yes. You're one of the best crowds I've worked with.

[3] Another traditional dance "Make a snake till it bites its tail" could be used also.
[4] Strictly the orbits consist of dancers, however as couple are always keep together we think of them as a unit. Really the men and woman form separate "parallel" orbits. Couples and dancers are used synonymously in this context.
[5] The actual paths are curved and looping as the dancer swing and move, these imaginary lines help to find the beginning and ending positions.

Would you please change your clothes, so that everyone in your circle now (i.e. your orbit) has the same color. I see there are 15 orbits, I'd like each orbit's color distinct, so I've got a list up here on stage of the 15 colors. Take 5 minutes to change. Oh, by the way chose one person in each of your circles to be number 0, and label yourselves with a consecutive orbit number as you move anticlockwise around your circle.

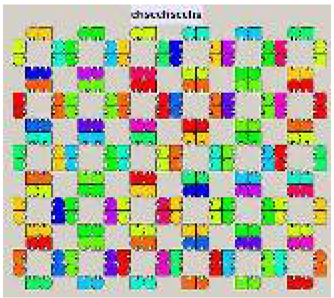


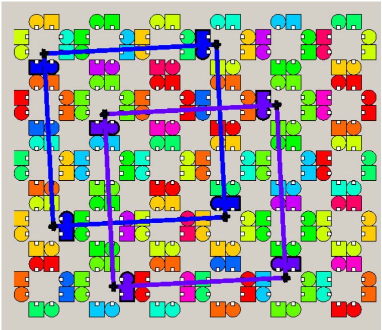
Figure 2

For 5 x 6 grid, N = 120 = 2^3x3x5 couples. In this example, d = chscchscchs, φ(d) = c, 15 Orbits: 6 of length 12, 3 of length 8, and 6 of length 4.
Period (i.e. order of d) is 24 = lcm(12, 8, 4).

Q: How do we look?

A: Great!

A: And you changed so quickly. I see 6 orbits of length 12, 3 orbits of length 8 and 6 orbits of length 4. Giving a total of 120 couples which of course what you'd expect as everyone is in exactly one orbit. Blue and purple orbits raise your hands. I can see you will come back to your spots every 4 times through the dance. Your orbits never share dancers in the same square, so none of you will dance with each other in this dance. Red orbit you've got the longest journey, every 12 times you return. Cyan you've got 8. I see you share quite a few squares with the red orbit, so there should be some interesting dancing between your two groups. For everyone to get back home it will take 24 times which is the least common multiple of all the orbit sizes.



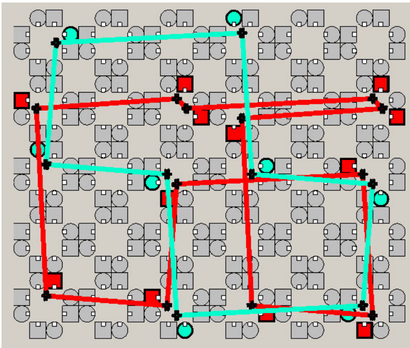


Figure 3

Q8 (Cyan women in bottom right square) I especially like my current corner, how often will have to wait till I see him again?
A: Since your orbit is length 8, and his is 12, it will take 24 times before you see him again in this same location, but because your orbits get close in other squares, you might get lucky (more details later.)

Q9 What is the complete cycle until everyone returns to their original spots?

A: The least common multiple of all the orbits sizes here 24. It is a multiple of 12, 8 and 4 and is the smallest such number.

Q10 Although I love this contra dance community, there are a few people I just don't want to dance with. How can I avoid them in a grid square?
A: Make sure you get in an orbit that doesn't share squares with theirs.

Q11: I'm number two in my red orbit, my friend in the same orbit is eight, will we dance together?
A: It's a big orbit, size 12, and you are far apart, it is unlikely that you will ever meet. However I see your brother is also in your orbit, and located in the number 3 spot. Inspecting the red orbit, I see that there are two places where consecutive red dancers are in the same square. So 2 out of every 12 times you will be dancing in the same square as your brother.

Q12: Since people keep returning to their home spots at various intervals, at any time though the dance a certain number of couples are in their home spots. Over the complete cycle of the dance, what is the average number such couples?
A: In this case 15. It is always the number of orbits.

A: Special case of the more general result on group action, although here with cyclic action it is easy to see directly. See Th 3.2.1 pg 49 Theory of Groups by Joseph J. Rotman (1965):

Let X be a finite set, and let G be a group of permutations on X. If K is the number of orbits of G, then $K = 1/|G| \sum_{g \text{ in } G} F(g)$ *where F(g) is the number of x in X fixed by g.*

Q: Can we dance it now?
A: Yes, good idea. Get ready for "**chscchscchs**" Band take it away! …

A: I must be confused or something. You didn't seem to be dancing because at the start of each time when I looked up from my dance card, you were exactly as before. I need you to dance it again later and get a better look.
Q: Don't you think you're being silly? Since we were dressed in orbit colors which are invariant under the movements of the dance, what did you expect? Maybe we can dance it again soon?

Summary: In contrast to a contra dance which has a simple periodic nature, a grid square has cyclic events that vary from dancer to dancer. The orbit structure of the dance permutation explains some of this. They explain why certain dancers will never meet.

Neighbors

A grid square has various types of neighbors: corners, opposites, anti-corners[6] and other people that you may interact in passing from one square to another. We use neighbors in the following sense[7]. Upon entering a square the various people relative to you in that square become your various type neighbors while you are in that square. So if you enter a square, the woman on the man's left is his corner while dancing in that square, the man on the woman's right is her corner, etc. So it needs to be taken from context when positions or dancers are being referred to. A pragmatic way to view this is partition the dancer's spots on the floor into pairs. In figure 1, pair up red men's spots with purple women's spots in each square, red woman with green men, green women with cyan men, etc. These are the corner pairings. When two dancers enter the square if they are in one of these pairs they will be called corners while in that square.

For those familiar with Bob Isaacs' Simple Pleasure, the dancers have a brief interaction with their opposite (the courtesy turn of a LC), an 8 beat swing with their anti-corner, and a partner swing.

We consider some period 2 regular permutations (thus decomposed into product of 2-cycles only) that are used to define a "neighbor." Most correspond to elements of SD.

- p = (12)(34)(56)(78) extended to GrSD for partner
- k = (81)(23)(45)(67) corner
- $f = c^2$ = (15)(26)(37)(48) flag pole opposite
- o = fp = pf = (16)(25)(38)(47) opposite
- a = fk = kf = (14)(27)(36)(58) anti-corner
- m = hsp, california move on neighbor

We have that f,p, and o commute with r (i.e. they are in centralizer(r)), and a, p, o, k, f are in centralizer(c).

Q13: Who is my neighbor (of type n = f, a, o, k or m) at time t?
A: $x\, d^t\, n\, d^{-t}$ which we denote N(x,t) when n is understood. So y = N(x,t) means $y\, d^t = x\, d^t\, n$ and $x\, d^t = y\, d^t\, n$.

Q14: Do those in my orbit have a similar neighbor experience?
A: Yes, $N(x\, d^k, t) = N(x, t+k)\, d^k$
A: Yes, Suppose as Liz moves through the grid, she has Bill, John, Mikki, Roger, Rob, Ian, … as corners. Now Heather who is one place ahead of Liz in their orbit won't expect to meet these same men, however she will meet similarly dressed men in

[6] Men have right hand ladies, we don't usually say women have left hand gents. Anti-corner is non-sexist.
[7] In contrast to some types of square dancing where the interpretation is relative to position. i.e. the person on the woman's right is her corner at that moment, or absolute in that one's original corner is the only person called the corner.

the same order as Liz. The dancers' color (orbit) and position labels help. Suppose Liz who is purple(3) sees these men: red(3), yellow(0), blue(3), red(10), blue(7), orange(1), … Then Heather who must be purple(4) will see yellow(1), blue(4), red(11), blue(0), orange(2), … Note the arithmetic must be done cyclically, here assuming the blue orbit has 8 couples numbered from 0, …, 7. So blue(7+1) becomes blue(0).

Q15: Will any of us purple women dance with the same real men?
A: Well if the yellow orbit is size 4, every fourth women in your orbit will dance with the same yellow men.

Q16: My orbit is short, is there any advantage to that?
A: If you get a neighbor with a long orbit, you will find you get to dance with lots of neighbors from the same orbit. For example if you have a size 4 orbit, and you dance with a size 12 orbit (purple say), you will dance with at least 12/4 = 3 different purple neighbors in 12 times thru the dance. It's not really size that matters, but being relatively prime helps.

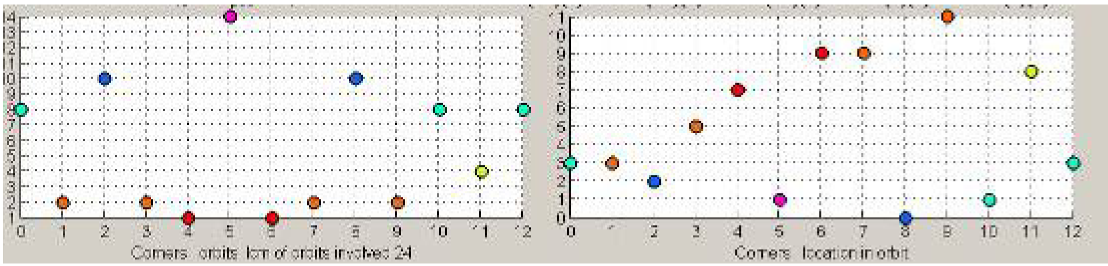
A*: If dancer A in orbit of size* ***a*** *meets a neighbor B with orbit size* ***b****, then A dances with at least lcm(****a,b****)/****a*** *different neighbors in B's orbit over the next lcm(****a,b****) times. These dancers are located at equal intervals around B's orbit at a spacing of gcd(****a,b****).*
Proof: Let **l** = lcm(**a,b**) and **g** = gcd(**a,b**). Recall **ab** = **lg**. Consider when and where this meeting occurs. Let t_0 be the time of first meeting B, and **i** be B's index in the orbit (so **i** in $\mathbf{Z}_b$). A will be at the same spot every **a** times at t = t_0, t_0+**a**, t_0+2**a**, … and thus meet the dancers in B's orbit with indices in order: **i**, **i**+**a**, **i**+2**a** … . In $\mathbf{Z}_b$, {**i**, **i**+**a**, **i**+2**a** …} = {**i**, **i**+ **g**, **i**+2**g**, … , **i**+(**l**/**a**-1)**g** } a set of size **l/a.** ■

Q17: Can you please give an example?
A: If your orbit size is 4 and the neighbors is 5, then during 20 times, you will dance with at least 20/4 = 5 (and hence all the 5 dancers).

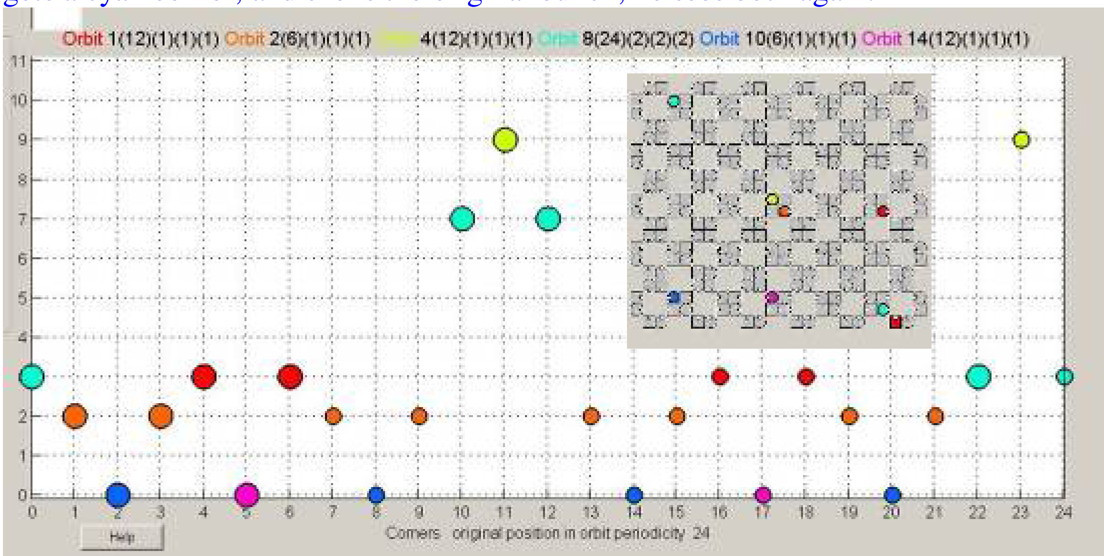
Q18: Can the previous result be strengthened?
A: Yes. The proof shows not only how many dancers in the orbit are met, but the specific dancers in order that will be met on the spot of original meeting. A careful inspection of A's orbit (viewed as the path on the floor) will allow more to be said. For example if there is one more spot where A meets a neighbor from B's orbit. Either that neighbor is not in the original bunch and A will dance with 2**l**/**a** people in B's orbit, or A will dance twice with each of the **l**/**a** folk. In this latter case it may happen the repeat dance[8] could happen as a sub cycle over **l**/2.

Here is corner tracking for the bottom red man in figure 3. Tracing his orbit and noting the orbit and position of the people standing there gives:

[8] See appendix for application to circle mixers and contra dances.



By shifting the points on the right graph above, he gets who he will have as corners, and the cyclic pattern of the meetings. In figure 3 red orbit is length 12, cyan is 8, he has a cyan corner initially. So over 24 times (lcm(8,12)) at times 0 and 12, he meets 2 (= 24/2) corners from the cyan orbit and they are spaced 4 apart in the cyan orbit. In figure 3 these are the bottom right and top left cyan woman. Since at t = 10, he also gets a cyan corner, and she is the original bunch, he sees both again.



So the dots of the same color on the graph all come from the basic interaction of the two orbits. The overall pattern is shown over the lcm of all the orbits met (24). Here two corners are from the cyan orbit (and are met twice), one from orange but met 8 times, one from blue met 4 times, one from yellow met twice, one from magenta met twice, and one from red (the same orbit as our dancer) met four times. Parts (with large dots) of the graph in one color are cyclic with a period dividing the lcm of the two orbits. It may happen that the overall graph has a cycle a divisor of lcm of all the orbits met.

You and your fellow orbiters experience the same dance: the same journey through the grid of squares, and the same sequence of meeting similar neighbors (i.e. same color clothes). Here we look at the relationships between a given pair of dancers. The simplest being of course you and your partner: throughout the dance you remain partners, but for two arbitrary people, it is conceivable a priori that they may meet up occasionally in various places as corners, and at other places as opposites. It turns out there are some constraints. Two people are *potential corners (opposites, anti-corners,* *etc.)* if at some times during the dance, they will be so. Keeping track of the times when this happens provides another aspect of the dance.

The dance permutation acts on pairs (doubleton sets). A pair (x,y) are *potential neighbors of type n* (for n = o, k, a, etc.) if there exists t such than y = N(x,t) which is equivalent to $yd^t = xd^t n$. This means they are potential neighbors if under the action on pairs, the resultant pair occupies **positions** in a square that are in this relationship. A *realization function* f where f(x,y)(t) is true at those t where y = N(x,t). |O(x,y)| = lcm(|O(x)||O(y)|). We can look at this for a particular neighbor type, or combine them to create a function which is true when there is some neighbor relationship realized.

Q19: Are there pairs of dancers that are potentially related?
A: Of course, in the initial set up of the grid squares all those pairs of people who are corners are potential corners (who happen to have this relationship realized). Similarly for current opposites, anti-corners, etc.

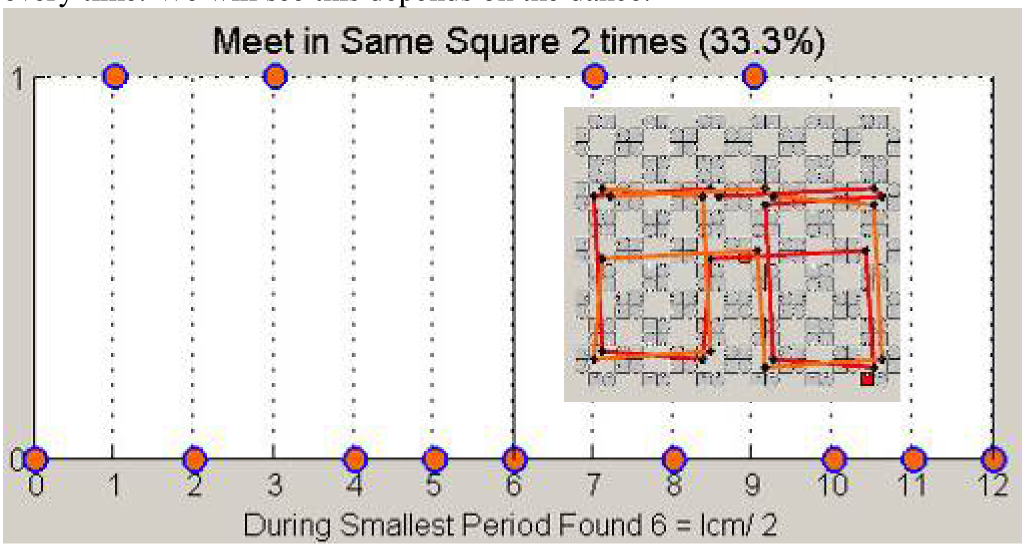
You've had a bit of a break. How about dancing it again? Any questions?

Q20 (red orbit(4) and cyan orbit(3)): I understand that me and my corner and all the other corner pairs standing together now are potential corners, but what other pairs are like this?
A: (red orbit(5), cyan orbit(4)) is such a pair. Just add the same value (using clock arithmetic) and you will find others. If all the corner pairs do this now, you will find all potential corners.

A: Each orbit of the action on pairs contains an initial neighbor pair.

Q: Could potential corners, be potential anti-corners?

A. The red and orange dancer below travel in orbits of same size (12), they meet in the same square 4 times in 12, in fact periodically 2 times in 6. They meet as corners every time. We will see this depends on the dance.



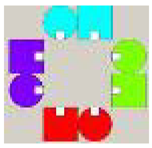
Q21 We have our colored spot labels. My spot color is cyan, and my corner's is green. How do they relate to this?
A: Oh I can see from what you've told me, you are a man.[9] You will find that your corners will all be cyan. The way the spot colors vary, depends on the particular dance. This dance is **chscchsccchs**. Sort the letters (backwards) to get "sshhhccccc", cross out any "h", "s", "rr" and "cccc" to get c. There can only be 8 possibilities: , c, cc, ccc, r, rc, rcc, rccc.

This table summarizes how the spot colors cycle around an orbit.

Type	Each Orbit contains spots of	Action on spot color
id	1 color	-- no change --
c	4 colors	(red, green, cyan, purple)
cc	2 colors: Red and cyan or green and purple	(red, cyan) (green, purple)
ccc	4 colors	(red, purple, cyan, green)
r	The 2 colors: red and cyan or 1 of other colors	(red, cyan)
rc	2 colors: red and purple or cyan and green	(red, purple) (cyan, green)
rcc	The 2 colors: green and purple or 1 of other colors	(green, purple)
rccc	2 colors: red and green or cyan and purple	(red, green) (cyan, purple)

This table has implications for orbit size. For example a "cc" type dance must have all orbits even, they would have to cycle through red, cyan, red, cyan, … or green and purple.

$\varphi(d)$ is one of the eight elements in CG and acts on $\{1,2,3,4\}$ which is identified to {red, green, cyan, purple}.

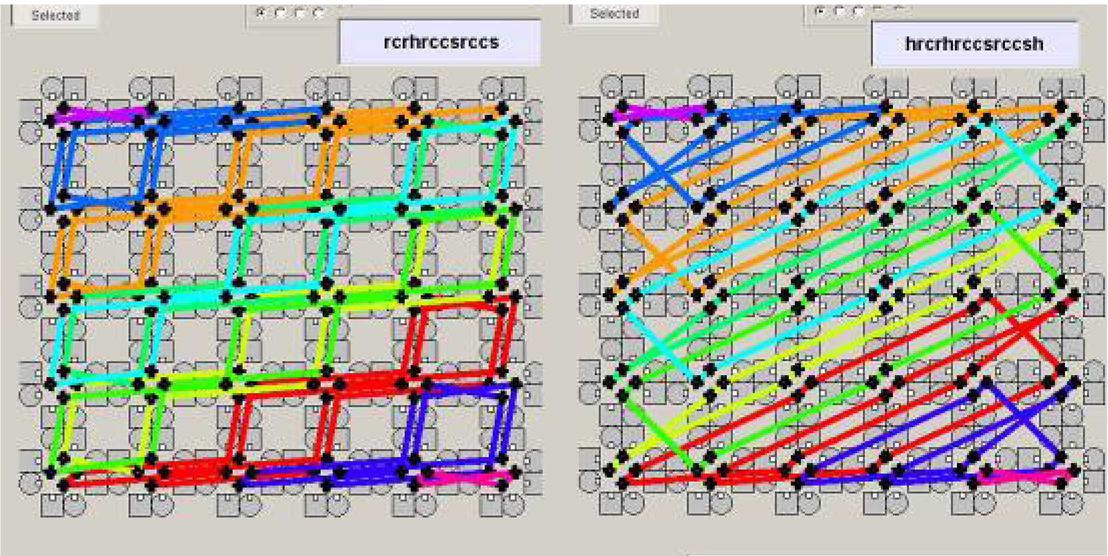


So for our dance you will notice that the spots cycle through (red, green, cyan, purple) as you move around any orbit. Looking at the spot colors in one square and observe that a (man, woman) pair are corners precisely when they are (red, purple), (green, red), (cyan, green) or (purple, cyan). In a similar way, anti-corners and opposites can be characterized by spot color combinations. Using the rule in the table to change both the first and second colors doesn't change this list. So for a "c" type dance, a given pair can potentially be only one kind of neighbor.

By checking out the cases, those dances without "r" in their type preserve corner, anti-corner and opposite potential relationships. That is if a pair is a potential corner, they will never be opposites or anti-corners, etc. For dances with an "r," potential corners can only be realized at even or odd times. Anti-corner relations are only possible at the other times, but none may be realized.

[9] In the original model square used for spot colors that is the only corner pairing with those colors, and the cyan person is the man.

Q22: As a mathematician and a caller, can I find other grid squares with a similar orbit structure?
A: Take any g and the conjugate gdg^{-1} has the same orbit structure, and hence theoretically an "isomorphic" dance. However there will be difficulties in practice. In the dance given by d, "balance and swing your corner" will become "balance and swing your *conjugate corner*[10]." This will be someone else in the grid, but they might not be in the same square as you, so rushing to swing them will be difficult. However we will see later that if g commutes with the permutations that define neighbor relationships, it will produce dances with a similar feel. Since k, o, a all commute with c, cdccc, ccdcc, and cccdc might be worth looking at, the relationships between the orbits is preserved too. Conjugating by other permutations, still give interesting dances. Example: d = rcrhrccsrccs and its conjugate hdh have the same orbit structure (10 Orbits of lengths 2x20, 6x12, 2x4), but for d, the green and yellow orbits share common squares, while not in the conjugate.



If the overall grid has "big flag pole" symmetry, i.e. the dance commutes with reflection through the very center, and conjugation by this reflection gives the identical dance.

Comments about orbit structure
For dances having an odd number of c's, dancers away from the edges of the grid tend to have small orbits of period 4. Dancers close to the edge belong to large orbit(s) that move around the edge.

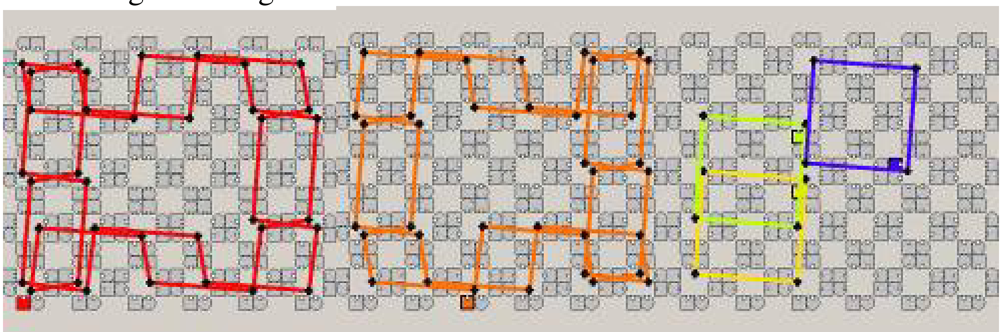
[10] Not to be confused with contra corners.

Extra Dancers around the edge

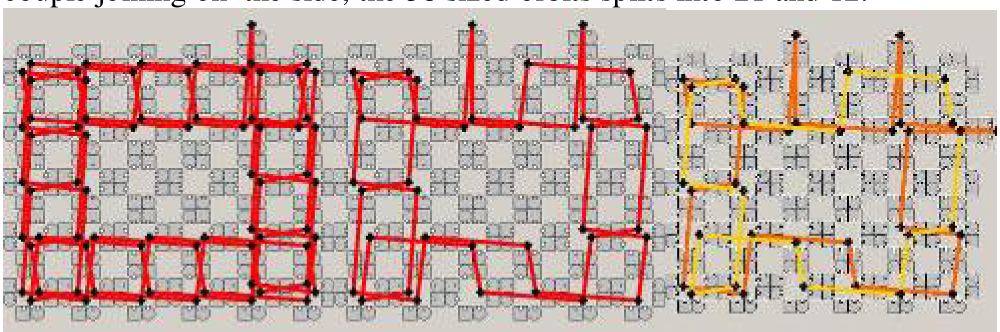
It is possible and even desirable when there are not a multiple of 8 couples, for extra couples to wait[11] around the edge of the grid, and come in. The spot coloring in this case is not longer invariant, and the orbit structure is more complicated (interesting☺). As dancers join around the outside, the larger orbits will include them, and sometimes break up into multiple orbits. The size of the orbits will usually have few common factors, the lcm determining periods is often large (in the 100's). Also φ fails to be well defined.

In practice since a dance will only run 16 to 20 times, it seems that the cyclic feel and neighbor meeting is similar to the first 16 to 20 of someone in the same dance without the added couples[12].

Here are some views of the orbits that change for ccchscchs (Big Can of Worms): In the rectangular 5x6 grid there are orbits: 2x36 outside and 12x4 inside.



As couples come it at the top, the two large orbits combine to one of size 73, with the next couple, an orbit of size 38 forms and one of 36 splits off again, with a third couple joining on the side, the 38 sized orbits splits into 21 and 12.



The small orbits of size 4 remain. The periods of the dance in the four examples are: 36, 292, 684, and 252.

[11] The generating permutations need to be defined for these dancers. All the ones generating SD fix or swap with partner (as these people are waiting). h and s are defined in the natural way. Mathematically these waiting dancers are their own corners etc.

[12] With the proviso that if you are close to the edge, the dance feels like the dance on full grid till you come off and wait, then when you come back on, it feels like the dance on the full grid for a possible different orbit.

Future exploration of how the orbit changes needs to be done, but the key will be how d is decomposed as a string of moves alternately in CG and h and/or s. For each h or s, dancers on head facing or side facing edges will reflect back in, and extra couples delay the movements as those on the outside will be missing out on some of the sequence.

Varying a dance sequence

In a square dance, it is customary to have breaks (that the dancers do with no walk thru) called between repetitions of the main figure. Also there may be variation as certain calls may be directed to heads or sides.

A break can be seen to have the effect of shifting some people to new orbits. The dances in the appendix have a “flip” version. This is a variant that reverses the circling aspect. For “c or ccc” type dances adding “cc” to the beginning of the dance description accomplishes this. In this way a mini-break of Circle ½ has been done. Note an actual break the rotates the square half way could be done instead.

Dancing with a partner, and moving to meet new people and dancing an easy repetitive pattern is one of the main aspects of a contra dance. So the emphasis on grid squares is to create a similar experience.

So the flips and variations might prevent people from getting stuck in a small loop (literally). A caller needs to be careful with dances (with and odd number of ‘c’) that have heads or sides doing the moving on. Alternately calling to heads and sides, customary for squares, is a bad idea for grids as the same people will be doing the moves each time. Also since small orbits are often size 4, flips or breaks should not occur at multiples of four.

(Expand and clarify) Sequences calling to Heads and Sides

Some dances have versions directed to H and S. (Call these dances H and S). For example one might use the sequence HSS … . This can be analysed as a repeated dance as follows. Consider the orbits and neighbors under the dance HSS. This gives the information at t = 0,3,6, … through the dance. Consider dancers after one H. Look at their orbits and neighbors under the dance SSH. The orbit gives the positions at t=1,4,7,…. and H^{-1} of the neighbors gives the neighbors at these times. Similarly consider dancers after HH, and use the dance SHS. Note that the dances HSS,SSH, and SHS are all conjugate and thus have the same orbit structure.

Visiting the grid but returning home

Although the goal is to move people around to maximize visiting, it is possible to engineer the dance using a simple break and the flip versions of a dance, so that after a predetermined time, all squares will be back with original couples[13]. Call "Simple Pleasures" 10 times, then a break which circles right ¼, then the flip of the dance 10 times. At this point all couples will all be back in home squares (a circle left ¼ from home spot). So a nice break with a promenade home would finish off the dance nicely. "Big Can of Worms" is similar, but a break of circle left ¼ is required.

The two examples above follow from this mathematical result: Consider dances of the following type $d=c^j m^k$ where m=cchs(=rhccrs) and j in $\mathbf{Z}_4$. (The permutation m stands for "heads and sides pass thru and move on.") Check $m^{-1}=c^2 m c^2$, so $m^{-k}=c^2 m^k c^2$. Let the flip of d be $f=c^{-j} m^k$ and the circling break $b=c^{j-2}$. Verify that $dbfb^{-1}=1$ and thus $d^n b f^n b^{-1}=1$. So calling the dance n times, the break b, the flip n times, and finally a break of b^{-1} brings everyone back to original positions. "Simple Pleasures" is of this type with j=3 and "Big Can of Worms" j=1.

Appendix:

Applications to other types of dance:

Circle mixers are dances where men and women form two orbits of the same length, n, say. If current partners shift k places, then a given person will meet lcm(n,k)/k people before returning to original partners. For example the "circle waltz" has k = 4. So with 18 couples everyone gets 9 different partners, with 19 couples 19 and with 20 couples 5.

Double progression contra dances can be viewed as just the head couples in a very simple grid (with imaginary sides). So the red cyan coloring applies to neighbors. With the number of couples n even, the progression k=2, a dancer can only see n/2 different neighbors. However if there is a couple out, then n is odd number and it is possible to meet all the other n-1 couples as neighbors[14].

[13] This is the goal of MWSD, and there the caller can vary the dance at will.
[14] Why not n? The n^{th} neighbor is your partner when you are out!

Dances

Some of these dances are on *youtube* on *nhcallers* channel. The permutation d is given, and the neighbors that get the most interaction. The way the dancers will move through the grid is only one (probably a minor) part. The way the moves fit together, the transitions between them are what make a dance interesting. For grid squares it is important that dancers stay oriented. This first dance is chs, but that is only the overall pattern, note that below the “c” part is the first 24 beats. Circling can be disguised - below A1,A2,B1 is a circle left ¼.

Grand Square Grid Bob Isaacs

May 2011
d=chs =cccm nbrs: n=o,p flip: ccchs

A1 Forward and Back; Four Ladies Chain
A2 Side Face Grand Square
B1 Grand Square (8 beats) Partner Swing
B2 Circle Left 1/2; California Twirl, Pass Thru (to new square)

Can Flip (F&B in B2). In B1, the sides walk straight into the swing, while the heads can start swinging when they meet in the center and swing into position.

Grand Square Grid (var) Bill Baritompa

May 2011
d=chs, n=o,p,m flip: ccchs

A1 RLG; Four Ladies Chain
A2 Side Face Grand Square
B1 Grand Square (8 beats) Partner Swing (face OUT)
B2 (from next square) Neighbor Balance and Swing
(face your partner now in the new square)
Flip is A1a Forward and Back

Simple Pleasures Bob Isaacs

Feb 2011
d=chs = cccm, n=o, a flip: ccchs

A1 Forward and Back; Four Ladies Chain
A2 Ladies Right Hand star 3/4, (pass your partner and next) Neighbor Swing
B1 Gent Left Hand Star 3/4, Partner Swing
B2 Circle Left 1/2; California Twirl, Pass Thru (to new square)
note: key is to know your partner to guide the stars

Twirl Through Bob Isaacs

Feb 2011
d=ccchs (this is a flip of Simple Pleasures) no swings, n = k interaction

A1 Corner Left Allemand, Grand Right and Left
A2 (Partner Dosido) Partner swing
B1 Balance, Petronella, balance, Petronella
B2 Balance, California twirl, pass through to new squares and forward and back

Balance the Grid Bob Isaacs

d=crhrh or cccrsccrs* n = k, o, a

A1 Forward and Back; Corner Left Allemand 1 ½
A2 (next) Corner Balance and Swing
B1 (next) Corner Left Allemand, RLG
B2 Right Allemand 1 ¼ (till)
Gents (connect and) RH* (1) (form the grid – interlocking waves ladies check)
C1 Balance the grid (R,L), AR (1/2), Balance (L,R), AL(1/2)
C2 Partner Balance and Swing
D1 Heads* Forward and Back; Heads Circle Left (1)
D2 Heads Zig left (4), right, left, right (moving 2 squares – if need be loop back)
ending: D1 Heads F&B; Side F&B D2: (all) Partner Balance and Swing
*note: in D1, D2 direct to Heads or Sides positions, so do not want to alternate as then the same people will always do it! H, S, S, H, S, S, H good as on youtube video!

Big Can of Worms Bob Isaacs

d=ccchscchs=cmm, n = k, o, a, m1, m2

A1 Forward and Back; Corner Swing
A2 Forward and Back; (next) Corner Swing
B1 (next) Corner Allemande Left, RLG
B2 Ladies Right Hand Star ½, Partner Swing
C1 Forward and Back; Circle left ½
C2 Balance the square, California Twirl;
(if see N in next square) Dosido 1 ½ (to move on)
otherwise with P California Twirl to face back (or nicer Swing)
D1, D2 same as C1, C2
ending: D2: Balance and Partner Swing
variation (which is flip) chscchs
B1: Corner Allemand left; Gents right hand star (1); B2: Partner Balance and Swing

Maze of Heys Bob Isaacs

d=crhrh or cccrsccrs*

A1 Forward and Back; Corner Swing
A2 Forward and Back; (next) Corner Swing
B1 Heads Right and Left thru; Sides Right and Left thru
B2 Corner Left Allemand, RLG
C1 Four LC (to Partner), Circle Left ½
C2 Heads* Zig left (4), right, left, right (moving 2 squares – if need be loop back)
D1 (in new squares) Heads Right Hand Star (1), Corner Allemand Left
D2 Partner Balance and Swing

version 2: B1=B2, B2=C1, C1=C2, so move 4 squares cccrhrhrhrh or crsccrsccrsccrs
*note: in C2, D1 direct to Heads or Sides positions, so do not want to alternate as then the same people will always do it! On youtube: v1: H,S,H v2:H,S,S,H,S

References[15]

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[15] Many helpful emails with Bob Isaacs. My approach was similar to that of Rachel, Amy, Bob and Rick while they were working on the paper to present at the Bridge conference. Rachel used my video of Simple Pleasures as part of her presentation.